Precautionary Saving: A Taxonomy of Prudence

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Abstract

We extend Gunning (2010) by analyzing the effect of risk on saving. We derive a general prudence index that determines the threshold for the future risk that makes saving increase. We relate our results to the utility premium of Friedman-Savage.

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1. Introduction

Precautionary saving is described as the extra saving generated by uncertainty regarding future income. This idea was first studied by Leland (1968) and Sandmo (1970), who showed that a positive third derivative of the utility function is required for positive precautionary saving. This condition is referred to as "prudence," a concept that was coined by Kimball (1990).

Precautionary saving is related to consumer attitudes toward risk. Given that the consumers are risk averse, uncertainty about future income causes disutility. Thus, if this disutility decreases in the consumption level, then consumers reallocate consumption from the present to the future in order to optimize intertemporal consumption. The disutility generated by future risk is reduced by precautionary saving (Eeckhoudt and Schlesinger, 2009).

The theoretical literature on saving recognizes that the effect of risk on savings depends not only on consumer attitudes toward risk but also on risk type. It is well known that the effect of labor income risk on savings is positive if the utility function exhibits prudence. This means that a consumer has a convex marginal utility function. However, when the source of uncertainty comes from other types of risk, this condition is no longer sufficient to guarantee the precautionary effect.

Eeckhoudt and Schlesinger (2008) examine the effect of risky interest rates on savings. Under an increase in second-degree risk, which Rothschild and Stiglitz (1970) define as a mean-preserving increase in risk, they show that a relative prudence over two guarantees the precautionary effect. Eeckhoudt and Schlesinger assume that consumers have access to a perfect capital market. However, households in developing countries do not have access to such a capital market and sometimes have to invest in projects with decreasing returns to capital income, such as on-farm investment (Gunning, 2010).

The purpose of this paper is to analyze the effects of four types of risk on saving: labor income risk, wealth risk, asset risk and capital income risk. To meet this objective, we follow a framework similar to Gunning (2010). However, unlike Gunning's formulation that is bases on a CRRA utility function, we assume that preferences satisfy standard properties, i.e, they are strictly increasing, strictly concave and exhibit prudence. We provide necessary and sufficient conditions on preferences to guarantee that a mean-preserving increase in risk guarantees a precautionary effect. A general prudence index is derived from conditions that determine the threshold for the future risk that makes saving increase. For the special cases of labor income risk and wealth risk, the general index is reduced to prudence and relative prudence, respectively. For the cases of asset risk and capital risk, the general index is reduced to the partial prudence index defined by Choi et al. (2001). We connect these results to the utility premium of Friedman and Savage (1948). In addition, we show that if the source of risk is the interest rate, as in Eeckhoudt and Schlesinger (2008), relative prudence is not the condition that defines the threshold. Instead, the partial prudence index, as defined by Choi et al. (2001), is the right choice. Gunning (2010)'s formulation then becomes a particular case of our model.

2. Risk and saving in a two-period model

We consider a simple two-period model with a sure income w in the first period but an uncertain income in the second. Let $F(\epsilon, r)$ denote the cumulative distribution function of a stochastic positive shock ϵ that effects the income in the second period, defined over a support within the closed interval $[a, b]$. The expectation of ϵ is defined as $E(\epsilon) = \int_a^b \epsilon f(\epsilon, r) d\epsilon$, where

$f(\epsilon, r)$ denotes the probability density function of ϵ, and r is a parameter that denotes a mean-preserving increase in risk.

The consumer has a strictly concave separable utility function $u()$ which is differentiable at least 3 times. The consumer seeks the optimal savings $k^*(r)$ that maximize the expected utility $U()$ of profits. Thus, the saving decision for a risk-averse consumer is:

$$U(k^*(r)) = \max_{k>0} U(k) \tag{1}$$

$$U(k) = u(c_1) + \beta E u(c_2) = u(c_1) + \beta \int_a^b u(c_2(k, \epsilon)) dF(\epsilon, r) \tag{2}$$

where $c_1 = w - k$ and $c_2 = c_2(\epsilon, k; y, \delta)$, y is expected labor income, δ is depreciation rate ($0 < \delta < 1$), and β is the discount factor ($0 < \beta < 1$). Following Gunning (2010), the wealth in the second period is given by:

$$c_2 = \epsilon_1 y + \epsilon_2 (1 - \delta) k + \epsilon_3 \mu(k) \tag{3}$$

where $(1 - \delta)k$ the expected value of assets, and $\mu(k)$ the expected value of capital income. The function $\mu(k)$ is increasing and concave, with $\mu(0) = 0$. We assume that $E(\epsilon_i) = 1$, with $i = 1, 2, 3$ and $E(c_2) = \mu_{c_2}$.

Whenever $\epsilon = \epsilon_1$, the shock is on the labor income, when $\epsilon = \epsilon_2$, the shock is in the assets, and if $\epsilon = \epsilon_3$, the shock is in the capital income. A shock to wealth is represented by $\epsilon = \epsilon_1 = \epsilon_2 = \epsilon_3$,

The first-order condition is given by:

$$U'(k^*(r)) = -u'(w - k^*(r)) + \beta E u'(c_2(k^*(r), \epsilon)) \frac{\partial c_2(k^*(r), \epsilon)}{\partial k} = 0 \tag{4}$$

We make use of the following theorem.

Theorem 1 (Diamond and Stiglitz, 1974)**.** *Let $\alpha^*(r)$ be the level of the control variable that maximizes $\int_a^b u(\theta, \alpha) dF(\theta, r)$. If increases in r represent mean-preserving increases in risk, then α^* increases (decreases) with r if U_α is strictly convex (concave) function of θ, i.e., if $u_{\alpha\theta\theta} > (<) 0$.*

We will use Diamond and Stiglitz's theorem to prove the following proposition.

Proposition 1. *Let $k^*(r)$ be the optimal level of saving that maximizes $U(k)$ in (2). Whenever r represents a mean-preserving increase in risk, then the level savings k^* increases (decreases) if and only if $g(\epsilon) = u'(c_2(k^*(r), \epsilon))\frac{\partial c_2(k^*(r),\epsilon)}{\partial k}$ is strictly convex (concave) in ϵ.*

Proof. Let $g(\epsilon) = u'(c_2(k^*(r), \epsilon))\frac{\partial c_2(k^*(r),\epsilon)}{\partial k}$ defined. Straightforward calculations show that:

$$g''(\epsilon) = (u'''(c_2)\frac{\partial c_2}{\partial \epsilon}\frac{\partial c_2}{\partial k} + 2u''(c_2)\frac{\partial^2 c_2}{\partial \epsilon \partial k})\frac{\partial c_2}{\partial \epsilon} \tag{5}$$

From Diamond and Stiglitz's theorem, is direct to show that k^* increases (decreases) if and only if $g''(\epsilon) > (<)0$. □

Note that $g''(\epsilon) > (<)0$ is equivalent to:

$$H(c_2) \equiv P(c_2)[\frac{\partial c_2}{\partial \epsilon}\frac{\partial c_2}{\partial k}/\frac{\partial^2 c_2}{\partial \epsilon \partial k}] > (<)2 \tag{6}$$

where $P(c_2) = -u'''(c_2)/u''(c_2)$ is absolute prudence. The expression $H(c_2)$ is a general prudence index, which takes different forms depending on the type of risk.

In order to determine the relationship between precautionary saving and the utility premium of Friedman and Savage (1948), we need to rewrite the consumer maximization problem as:

$$U(k^*(r)) = \max_{k>0} U(k) \tag{7}$$

$$U(k) = u(c_1) + \beta(u(\mu_{c_2}) - \pi(c_2)) \tag{8}$$

where $\pi(c_2) = u(\mu_{c_2}) - E(u(c_2))$ is the utility premium. The utility premium simply measures the loss of utility from consuming the random quantity c_2 instead of the certain quantity μ_{c_2}. Eeckhoudt and Schlesinger (2009) refer to this utility premium as an intrapersonal measure of pain, where pain is measured via a decrease in expected utility.

The first-order condition is given by:

$$u'(c_1) = \beta(u'(\mu_{c_2})\frac{\partial \mu_{c_2}}{\partial k} - \pi'(c_2)) \tag{9}$$

where $\pi'(c_2) = u'(\mu_{c_2})\partial\mu_{c_2}/\partial k - Eu'(c_2)\partial c_2/\partial k$

We note that the equation (9) is equivalent to (4). This condition shows that there are two reasons to reallocate one unit of wealth from period 1 to period 2: an intertemporal motive and a precautionary motive. The first is to smooth intertemporal consumption, while the second is to reduce the pain associated with risk future. The second reason depends on the sign of $\pi'(c_2)$. From (9), we have that $k^* > (<)\hat{k}$ if and only if $\pi'(c_2) < (>)0$, where $\hat{k}$ is the saving under certainty. This means that a consumer can reduce the pain of future risk by shifting a bit more wealth from period 1 to period 2 if the utility premium is decreasing in saving. Precautionary saving is additional saving that reduces the pain of future risk. Note that from the definition of the utility premium, we have $\pi'(c_2) < (>)0$ if and only if:

$$Eu'(c_2)\frac{\partial c_2}{\partial k} > (<)u'(\mu_{c_2})\frac{\partial \mu_{c_2}}{\partial k} \tag{10}$$

By Jensen's inequality, (10) will hold if the function $g(\epsilon) = u'(c_2)\partial c_2/\partial k$ is strictly convex (concave) in ϵ. This result is precisely the one obtained above in proposition 1.

We will now see the forms taken by condition (6) for the four types of risk.

2.1. Labor income risk

In this case, equation (3) is:

$$c_2 = \epsilon y + (1-\delta)k + \mu(k) \tag{11}$$

We show straightforwardly that:

$$\frac{\partial c_2}{\partial k} = (1-\delta) + \mu'(k); \frac{\partial c_2}{\partial \epsilon} = y; \frac{\partial^2 c_2}{\partial \epsilon \partial k} = 0 \tag{12}$$

Replacing (12) in (6), we have:

$$P(c_2)[y(1-\delta) + y\mu'(k)] > (<)0 \tag{13}$$

Given the assumptions of the model, condition (13) is positive (negative) whenever $u'''(c_2) > (<)0$. Thus, prudence ($u''' > 0$) guarantees that an increase in labor income risk increases saving in the present.

We note that the CRRA assumption implies that $u(c_2) = R(R+1)c_2^{-(R+2)}$. Therefore, as established in Gunning (2010), the effect of risk on saving is positive for all $R > 0$.

2.2. Wealth risk

Whenever the shock is in wealth, equation (3) is transformed to:

$$c_2 = \epsilon(y + (1-\delta)k + \mu(k)) \tag{14}$$

It is directly shown that:

$$\frac{\partial c_2}{\partial k} = \epsilon((1-\delta)+\mu'(k)); \frac{\partial c_2}{\partial \epsilon} = y+(1-\delta)k+\mu(k); \frac{\partial^2 c_2}{\partial \epsilon \partial k} = (1-\delta)+\mu'(k) \tag{15}$$

Replacing (15) in (6) and rearranging the expression, we have:

$$c_2 P(c_2) > (<)2 \tag{16}$$

The left-hand side of (16) is known as relative prudence, i.e., the coefficient of absolute prudence $P(c_2)$ multiplied by the level of wealth destined for consumption in the second period c_2.

For the special case of the CRRA utility function, relative prudence is $R+1$. From (16), we have that the precautionary effect is positive (negative) if and only if $R > (<)1$.

Menezes and Hanson (1970) and Zeckhauser and Keeler (1970) introduced an index of partial relative risk aversion defined as $A_p(b, y) = -(y - b)u''(y)/u'(y)$. In a similar way, Choi et al. (2001) introduced an index of partial prudence as $P_p(b, y) = -(y - b)u'''(y)/u''(y)$. We will now see that asset risk and capital income risk are special cases of the index defined by Choi.

2.3. Assets risk and capital income risk

In these cases, condition (6) is reduced to:

$$(c_2 - b)P(c_2) > (<)2 \tag{17}$$

where $b = y+\mu(k)+\mu'(k)k$ to asset risk, while $b = y+(1-\delta)(k-\mu(k)/\mu'(k))$ to capital income risk. Therefore, saving increases (decreases) if and only if the measure of partial prudence exceeds (is less than) 2. For the case of the CRRA function, condition (17) is reduced to:

$$(\frac{c_2 - b}{c_2})(R + 1) > (<)2 \tag{18}$$

We note that for $\delta = 1$ and $\mu(k) = rk$, consumption period 2 is $c_2 = y+\epsilon rk$. In this case, condition (6) is reduced to $P(c_2)(\epsilon kr) > (<)2$. Whenever $b = y$, then $(c_2 - y)P(c_2) > (<)2$. However, for $y = 0$, the condition for that precautionary saving occurs if relative prudence exists, which is the case of interest-rate risk analyzed in Eeckhoudt and Schlesinger (2008).

3. Conclusion

This paper is an extension of Gunning (2010). We examined the effects of four types of risk on precautionary saving. We derive a general prudence index that determines the threshold for the future risk that makes saving increase (decrease). For the special case of labor income risk, the general index is reduced to prudence. However, when the source of uncertainty comes from wealth risk, asset risk or capital income risk, prudence is no longer sufficient to guarantee the precautionary effect. In the case of wealth risk, if relative prudence exceeds 2, it guarantees that precautionary saving occurs. In the cases of asset risk and capital income risk, partial prudence exceeding 2 is a condition that guarantees the precautionary effect.

These results are related to the utility premium of Friedman and Savage. The precautionary effect arises if and only if the utility premium is decreasing in saving. This means that a consumer can reduce the pain of future risk by shifting a bit more wealth from period 1 to period 2. Precautionary saving is the additional saving that reduces the pain of future risk.

4. References

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